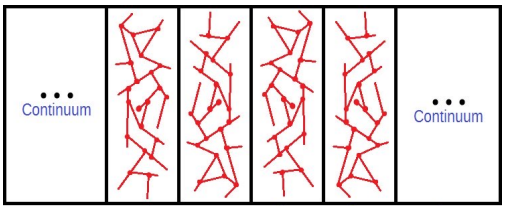
Topics in ERGODIC THEORY and MEASURED GROUP THEORY

MATH 594/740 Anush Tserunyan 2022 WINTER

Location: Zoom link TueThu 11:35am–12:55pm

Ergodic theory studies actions of countable (semi)groups on a measure space (typically, probability space), while measured group theory studies countable groups by studying their actions on probability spaces, so the two subjects are closely tied but their paradigms are different. The course will introduce these subjects and cover selected topics from them. Besides the topics themselves, the main goal of the course is to instill the vision and methods that descriptive set theory offers to the study of measurable actions of countable groups by reducing statements about them to graph combinatorics. The interaction of descriptive set theory and various other subjects has been a fruitful and rapidly expanding area of research in the past 30 years and this course will give a glimpse of it. As an example, in half a lecture, we will learn a new combinatorial proof of the classical pointwise ergodic theorem using nothing but definitions.



A graphing of a countable group action

A tentative list of topics from ergodic theory includes ergodicity and pointwise ergodic theorems, various notions of mixing, entropy and the classification of dynamical systems up to isomorphism; while in measured group theory, we will learn the orbit equivalence (OE) of group actions and classification of actions up to OE, the notion of cost, and we will prove D. Gaboriau's fundamental theorem of cost, which allows to distinguish actions up to OE.

Prerequisites: Comfort with

- basic *pointset topology* and *metric spaces* (open/closed sets, convergence, compactness, continuity, product topology),
- *abstract measure theory* (Borel and measurable sets and functions, abstract measures and product measures, L^1-functions),
- *group actions* (definition, orbits),
- and *graph-theoretic terminology* (paths/cycles, connectedness, trees).

No knowledge of descriptive set theory is required, we will cover what we need on the fly.

Coursework: 2-3 homework assignments: solutions are presented at the board by the students and the presentations are evaluated, but no write-ups are necessary.

Method of evaluation: The grade will be loosely determined by class participation and homework solution presentations.